Computer Architecture

Sheet (6)

6.1 Consider the binary numbers in the following addition and subtraction problems to be signed, 6-bit values in the 2's-complement representation. Perform the operations indicated, specify whether or not arithmetic overflow occurs, and check your answers by converting operands and results to decimal sign-and-magnitude representation.

$$
\begin{array}{cc}
110111 & 010101 \\
+111001 & +101011 \\
\hline
\end{array}
$$

$$
\begin{array}{cc}
111110 & 100001 \\
-100101 & -011101 \\
\hline
\end{array}
$$

$$
\begin{array}{cc}
000111 & 011010 \\
-111000 & -100010 \\
\hline
\end{array}
$$

6.9 Show that the logic expression $c_n \oplus c_{n-1}$ is a correct indicator of overflow in the addition of 2's-complement integers, by using an appropriate truth table.

6.10 (a) Design a 64-bit adder that uses four of the 16-bit carry-lookahead adders shown in Figure 6.5 along with additional logic to generate c_{16}, c_{32}, c_{48}, and c_{64}, from c_0 and the G_i^{II} and P_i^{II} variables shown in this figure. What is the relationship of the additional logic to the logic inside each lookahead circuit in the figure?

(b) Show that the delay through the 64-bit adder is 12 gate delays for s_{63} and 7 gate delays for c_{64}, as claimed at the end of Section 6.2.1.

(c) Compare the gate delays to produce s_{31} and c_{32} in the 64-bit adder of part (a) to the gate delays for the same variables in the 32-bit adder built from a cascade of two 16-bit adders, as discussed in Section 6.2.1.

6.11 (a) How many logic gates are needed to build the 4-bit carry-lookahead adder shown in Figure 6.4?

(b) Use appropriate parts of the result from Part (a) to calculate how many logic gates are needed to build the 16-bit carry-lookahead adder shown in Figure 6.5.

6.12 Show that the worst case delay through an $n \times n$ array of the type shown in Figure 6.6b is $6(n-1) - 1$ gate delays, as claimed in Section 6.3.

6.17 Multiply each of the following pairs of signed 2's-complement numbers using the Booth algorithm. In each case, assume that A is the multiplicand and B is the multiplier.

(a) $A = 010111$ and $B = 110110$

(b) $A = 110011$ and $B = 101100$

(c) $A = 110101$ and $B = 011011$

(d) $A = 001111$ and $B = 001111$

6.18 Repeat Problem 6.17 using bit-pairing of the multipliers.

6.19 Indicate generally how to modify the circuit diagram in Figure 6.7a to implement multiplication of signed, 2's-complement, n-bit numbers using the Booth algorithm, by clearly specifying inputs and outputs for the Control sequencer and any other changes needed around the adder and A register.

*) Using non-restoring division perform the operation A % B on the five bit numbers A=10101 and B=00101.